Automatic programming via large language models with population self-evolution for dynamic job shop scheduling problem

Jin Huang, Xinyu Li, *Member, IEEE,* Liang Gao, *Senior Member, IEEE,* Qihao Liu, Yue Teng

***Abstract*—Heuristic dispatching rules (HDRs) are widely regarded as effective methods for solving dynamic job shop scheduling problems (DJSSP) in real-world production environments. However, their performance is highly scenario-dependent, often requiring expert customization. To address this, genetic programming (GP) and gene expression programming (GEP) have been extensively used for automatic algorithm design. Nevertheless, these approaches often face challenges due to high randomness in the search process and limited generalization ability, hindering the application of trained dispatching rules to new scenarios or dynamic environments. Recently, the integration of large language models (LLMs) with evolutionary algorithms has opened new avenues for prompt engineering and automatic algorithm design. To enhance the capabilities of LLMs in automatic HDRs design, this paper proposes a novel population self-evolutionary (SeEvo) method, a general search framework inspired by the self-reflective design strategies of human experts. The SeEvo method accelerates the search process and enhances exploration capabilities. Experimental results show that the proposed SeEvo method outperforms GP, GEP, end-to-end deep reinforcement learning methods, and more than 10 common HDRs from the literature, particularly in unseen and dynamic scenarios.**

***Index Terms*—dynamic job shop scheduling problems (DJSSP), large language models (LLMs), automatic heuristic dispatching rules design, self-evolutionary (SeEvo)**

I. Introduction

THE core challenge of production scheduling lies in the efficient allocation of limited resources, such as machinery, to ensure the completion of tasks within the planning horizon while optimizing predefined performance metrics [1], [2]. The job shop scheduling problem (JSSP), recognized as an NP-hard optimization problem, is typically addressed using traditional exact algorithms like dynamic programming [3] and branch-and-bound [4]. However, these methods are unsuitable for large-scale cases [5], [6]. Consequently, researchers have turned to metaheuristic algorithms to achieve near-optimal solutions for large-scale static cases within acceptable computational time [7], [8]. In dynamic job shop scheduling problems (DJSSP), real-time changes such as randomly arriving orders and machine breakdowns create additional challenges. Heuristic dispatching rules (HDRs) have proven effective in such dynamic environments due to their low computational complexity and ability to respond quickly to changes [9], [10].

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Efficiently designing HDRs is crucial for improving manufacturing system performance. Despite the development of various heuristic methods, most require extensive expert knowledge and fine-tuning. To overcome this, the automated generation of HDRs has become a research focus, with genetic programming (GP) [11] and gene expression programming (GEP) [5] being two widely used approaches for automatic algorithm design.

GP simulates natural selection and genetic mechanisms to automatically generate and evolve complex HDRs. However, the traditional tree-based representation of GP can lead to an overly complex search space and increased computational costs [12]. To address these limitations, GEP is proposed as an alternative, using fixed-length linear chromosomes that simplify genetic operations and improve the manageability of evolved solutions [1].

Despite the success of GP and GEP in automatic algorithm design, both methods face limitations, particularly in their generalization performance when applied to unseen DJSSP cases, where deep reinforcement learning (DRL) often performs better [13]. Moreover, both methods lack the capability for effective self-guided exploration, relying heavily on random searches, which limits the algorithms' exploration and exploitation efficiency.

However, although DRL-based methods demonstrate notable generalization capabilities, their scheduling performance still falls short of optimal. In some cases, these methods provide no substantial improvement over HDRs [14], [15]. This limitation has led researchers to explore integrated approaches, incorporating GP-based action spaces within DRL frameworks [16], as well as end-to-end DRL strategies grounded in graph neural networks [17], [18]. These innovations offer a partial improvement in exploration and exploitation efficiency, enhancing DRL's adaptability and effectiveness in dynamic scheduling environments.

Recently, the rise of large language models (LLMs) has introduced new possibilities for integrating evolutionary algorithms with automatic algorithm design [19]. Through prompt engineering and iterative feedback, LLMs generate highly adaptive and targeted heuristic rules by leveraging vast domain knowledge and pattern recognition [20]. Studies such as applying LLMs in online bin packing problems, published in *Nature* [21], highlight the potential of this approach. However, the application of LLMs to more complex problems, such as DJSSP, remains relatively unexplored. Although frameworks like ReEvo have successfully applied in the traveling salesman problem (TSP), the dynamic and variable nature of DJSSP introduces extra challenges that require more specialized and flexible approaches.

Therefore, this paper proposes a novel LLM-based evolutionary framework for automatic algorithm design in DJSSP, introducing an innovative population self-evolutionary (SeEvo) method. This method leverages LLMs as hyper-heuristic generators and employs self-evolution within a population to automatically design and optimize HDRs. Specifically, the contributions of this paper are as follows:

1) **Novel LLM-Based Evolutionary Framework for DJSSP:** This paper introduces a novel LLM-based evolutionary framework that addresses the generalization limitations and stochasticity inherent in the automatic design of algorithms for DJSSP. The framework leverages LLMs to generate adaptive heuristic rules, enhancing the automatic algorithm design.
2) **Population Self-Evolution Strategy:** A novel population self-evolution strategy is proposed within the LLM-based framework, significantly improving the exploration and exploitation capabilities of the generated heuristics. This strategy allows for continuous refinement of HDRs based on real-time feedback during the scheduling process, enhancing scheduling efficiency in dynamic environments.
3) **Comprehensive Evaluation and Superiority of the SeEvo Method:** The proposed SeEvo method's effectiveness and superiority are demonstrated through extensive comparisons with commonly used HDRs, GP, GEP, and end-to-end DRL methods. Experimental results demonstrate the SeEvo method's superior generalization across unseen and dynamic DJSSP, outperforming other dynamic scheduling methods.

The remainder of this paper is organized as follows. Section II provides the problem formulation of DJSSP and reviews relevant literature on job shop scheduling methods and automatic algorithm design with large language models. Section III introduces the proposed language-heuristic-based DJSSP framework, followed by a detailed explanation of the population self-evolution method in Section IV. Experimental setup and performance evaluation are presented in Section V. Finally, Section VI offers conclusions for this paper.

II. Background

A. Mathematical model of Job Shop Scheduling

The DJSSP is an extension of the classical JSP to accommodate dynamic changes, such as random job arrivals, machine breakdowns, and varying processing times [22]. This study specifically focuses on the dynamic event of random job order arrivals. The objective is to develop a scheduling policy that minimizes the makespan $C_{\max}$, while simultaneously adapting to real-time fluctuations in the system [23], [24].

Problem Definition:

- $K = \{1, 2, ..., m\}$: A set of m machines.
- $I = \{1, 2, ..., n\}$: A set of n jobs, where each job $i \in I$.
- $K_{ij} \subseteq K$: The set of possible machines on which the operation O_{ij} can be processed.
- p_{ik}: The processing time of job O_i on machine k.
- t_{arr_i}: The arrival time of job i.

Decision Variables:

- x_{ik}: Start time of job i on machine k.
- $z_{ii'k} \in \{0, 1\}$: Binary decision variable. $z_{ii'k} = 1$ if job i is processed before job i' on machine k; otherwise, $z_{ii'k} = 0$.
- $C_{\max}$: The makespan, defined as the maximum completion time among all jobs.

Objective Function:

$$\text{Minimize } C_{\max} \tag{1}$$

Constraints:

$$x_{iK_{ij}} \geq x_{iK_{ij-1}} + p_{iK_{ij-1}}, \forall j \in \{2, 3, ..., q_i\}, i \in I \tag{2}$$

$$x_{i'k} \geq x_{ik} + p_{ik} - Mz_{ii'k}, \forall i, i' \in I, i < i', k \in K \tag{3}$$

$$x_{ik} \geq x_{i'k} + p_{i'k} - M(1 - z_{ii'k}), \forall i, i' \in I, i < i', k \in K \tag{4}$$

$$C_{\max} \geq x_{iK_{im}} + p_{iK_{im}}, \forall i \in I \tag{5}$$

$$x_{ik} \geq t_{\text{arr}_i}, \forall i \in I, k \in K \tag{6}$$

$$z_{ii'k} \in \{0, 1\}, \forall i, i' \in I, k \in K \tag{7}$$

where Constraint (1) defines the objective function, which aims to minimize the maximum completion time. Constraint (2) ensures that the operations of the same job are processed in the predefined sequence. Constraints (3) and (4) guarantee that at any given time, a machine can only process one operation. Constraint (5) calculates the maximum completion time. Constraints (6) and (7) define the range of the two decision variables, with the initial arrival time for all jobs set to $t_{\text{arr}_i} = 0$.

B. Related Works of Job Shop Scheduling

Over the past few decades, numerous approaches have been proposed to address the JSSP. Exact algorithms, such as dynamic programming [3] and branch-and-bound [4], can find optimal solutions but are limited to smaller cases due to their computational complexity [4], [25]. To overcome these limitations, meta-heuristic methods, including genetic algorithm [26], [27], particle swarm optimization algorithm [28], [29], and memetic algorithm [30], have been widely employed to provide near-optimal solutions for larger-scale problems. However, these methods encounter challenges in dynamic job shop environments where problem conditions change frequently, often failing to generate high-quality solutions within reasonable computational times.

Heuristic dispatching rules (HDRs), which prioritize jobs or machines based on simple rules, are widely adopted in dynamic environments due to their efficiency and responsiveness to real-time changes [31]. While effective, the performance of these heuristic methods is highly scenario-dependent, often requiring expert customization. Among these, GP [32] and GEP [5], as types of hyper-heuristic methods, have shown particular promise in DJSSP scenarios. They can generate scheduling heuristics without domain-specific knowledge, providing an automated and adaptive means of solving complex scheduling problems [1], [33], [11]. However, one of the major challenges for these hyper-heuristic algorithms is the expansive feature space in DJSSP, which can enlarge the search space and limit exploration efficiency [34]. Moreover, heuristics generated by these methods may struggle to generalize well to unseen DJSSP cases, making it challenging to achieve high-quality dynamic scheduling solutions.

In recent years, DRL has been extensively explored as an alternative to address the generalization limitations of GP and GEP in DJSSP scenarios [35]. DRL-based methods allow agents to interact with the job shop environment, learning policies that maximize long-term rewards [36]. These methods can generate scheduling actions at decision points based on real-time state information and uncertainty, allowing for dynamic adjustments as new tasks arise. Trained policies can generalize across cases of varying sizes, enabling scalability [15]. However, although the scheduling policies in these DRL-based methods are size-invariant, the performance of the scheduling agents remains far from optimal, with some methods offering no advantage over individual HDR [14], [15]. Therefore, research has integrated GP-based action spaces into DRL frameworks, yielding better results compared to standalone HDRs [16]. Additionally, end-to-end DRL methods [17], [18], similar to the language-heuristic approaches proposed in this work, have been explored for directly selecting workpieces in DJSSP environments. These methods leverage graph neural networks to extract features and select workpieces in an end-to-end manner, achieving promising results in static generalization scenarios. Nonetheless, in dynamic cases, their performance still often falls short compared to common PDRs[17], [18].

C. Automatic Algorithm Design with LLMs

The rise of LLMs has opened new avenues for automatic algorithm design through their integration with evolutionary algorithms. LLMs have demonstrated substantial capabilities in tasks such as code generation [37], code optimization [38], [39], solving algorithmic competition challenges [40], [41], generated data [42], and robotics control [43]. These advancements have extended to areas such as prompt optimization [44], reinforcement learning reward design [45], and algorithm self-improvement [46]. Specifically, the integration of LLMs with evolutionary algorithms has shown great promise in solving combinatorial optimization problems such as the TSP [47], [46] and the online bin packing problem (BPP) [21].

While LLMs have proven effective in developing heuristic algorithms for simpler combinatorial problems like TSP and BPP, their application to more complex problems such as DJSSP remains relatively unexplored. One notable advancement is the ReEvo framework [20], a language-heuristic approach that has achieved success in solving TSP by simulating the reflective processes of human experts. However, the complexity of DJSSP surpasses that of TSP, as DJSSP involves scheduling multiple jobs across various machines, with dynamic processing times, making it significantly more challenging. Although the ReEvo framework has demonstrated excellent performance, particularly with its short-term (comparative learning of individual differences) and long-term reflection (summarization of short-term reflections) mechanisms, it has not fully explored LLMs' potential for self-evolution during individual iterations. Additionally, ReEvo was designed for static problems and lacked differentiation between train sets and test sets.

Given DJSSP's dynamic and variable nature, with frequent changes in processing scenarios, this paper seeks to introduce a novel language-heuristic framework that harnesses LLMs' full capabilities for automatic algorithm design. We aim to provide more efficient and generalizable solutions for this problem by leveraging LLMs' ability to generate adaptable heuristics.

III. Framework of Language-Heuristic-Based DJSSP

The proposed framework for DJSSP, as shown in Fig.1, consists of two main phases: the self-evolution phase and the online application phase. To handle the DJSSP with randomly arriving orders, a job shop simulation environment is designed, coupled with a language-heuristic-based SeEvo method that evolves HDRs automatically.

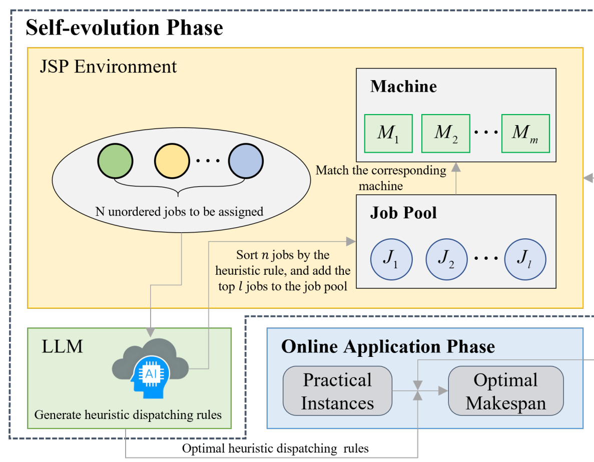

Fig. 1. Language-heuristic-based DJSSP framework.

As illustrated in Fig.1, during the self-evolution phase, the DJSSP environment comprises a job pool and a set of machines. Incoming jobs are first fed into the LLM, which generates heuristic rules based on the statistical properties of the dataset. These rules are used to prioritize and sequence jobs, with only the top-ranked jobs available for immediate scheduling. After completing each job, the machine becomes free and selects the next job from the job pool. Throughout this phase, the LLM collects extensive training data and iteratively improves the HDRs to optimize the scheduling performance.

This phase primarily serves as a training period, during which multiple cases are processed to refine the HDRs. In the subsequent online application phase, the HDRs and prompts obtained from training on the individual cases during the self-evolution phase (20 cases in our experiments) are each applied in practical scenarios. With well-designed HDRs and prompts, the system can rapidly generate high-quality HDRs after a single iteration of the framework. For instance, in tests involving 50 jobs and 15 machines, the framework delivers a high-quality solution within 30 seconds.

To address challenges associated with generating high-quality HDRs during the self-evolution phase, this paper proposes a novel SeEvo method that enhances exploration through individual co-evolution reflection, individual self-evolution reflection, and centralized evolution reflection strategies. Detailed explanations of these mechanisms are provided in the following subsections.

IV. Language-Heuristic-Based Population Self-Evolution Method

The overall structure of the language-heuristic-based population SeEvo method is depicted in Fig.2. In this framework, LLMs play two essential roles: generating guiding prompts for the population and creating individual heuristic programs. Unlike conventional hyper-heuristic approaches, SeEvo relies on the independent generation of heuristic code segments, which are continuously refined and optimized throughout the evolution process. The key to SeEvo's success lies in leveraging LLMs to generate initial guiding prompts and craft individual heuristic programs tailored to the scheduling tasks. The detailed procedure of the SeEvo evolutionary process is presented in Algorithm 1.

Algorithm 1: SeEvo Evolutionary Process

Input: Population $\mathcal{P}$, Function Evaluations FE, Maximum Function Evaluations $maxFE$, Elitist e, Case Number N, Mutate Probability P_m

Output: Best Code, Best Code Path

while $FE < maxFE$ **do**
  if *all individuals are invalid* **then**
    Raise Error: "All individuals are invalid";
  end
  $S_p \leftarrow$ Select population from $\mathcal{P}$;
  if S_p *is None* **then**
    Raise Error: "Selection Failed";
  end
  $S_R \leftarrow$ Individual Co-Evolution Reflection(S_p);
  $P_{inter} \leftarrow$ Crossover(S_R, S_p);
  $\mathcal{P} \leftarrow$ Evaluate Population(P_{inter}, N);
  e, Best Code, Best Code Path $\leftarrow$ Update Iteration;
  $I_R \leftarrow$ Individual Self-Evolution Reflection($\mathcal{P}, R, S_p$);
  $P_{self} \leftarrow$ Crossover($I_R, \mathcal{P}$);
  $\mathcal{P} \leftarrow$ Evaluate Population(P_{self}, N);
  e, Best Code, Best Code Path $\leftarrow$ Update Iteration;
  $M_R \leftarrow$ Collective Evolution Reflection(P_m, S_R);
  $P_{mut} \leftarrow$ Mutate(P_m, M_R);
  $\mathcal{P} \leftarrow \mathcal{P} \cup$ Evaluate Population(P_{mut}, N);
  e, Best Code, Best Code Path $\leftarrow$ Update Iteration;
  $FE \leftarrow FE + 1$;
end
Return Best Code, Best Code Path;

SeEvo proceeds through eight principal stages, each contributing to refining the population of heuristics to improve job selection efficacy. Below is an overview of these eight key steps:

Individual Encoding: The SeEvo method continuously evolves heuristic algorithms, but its encoding mechanism differs from traditional evolutionary algorithms. SeEvo individuals are heuristic code segments designed to guide job selection in DJSSP rather than directly determining the final scheduling plan. Furthermore, these individuals are generated by the LLM without predefined constraints on encoding length or function sets, which are common limitations in traditional algorithms like GEP. The only requirement for the LLM-generated code is to adhere to the specified function names, input parameters, and output parameters.

Initialization: In the SeEvo method, population initialization is carried out using the LLM prompt generator, which takes task specifications and seed heuristics as inputs. The task specifications provide details about the JSSP, input parameters, and heuristic functions. Seed heuristics (example heuristic codes) serve as a foundation, guiding the LLM to generate initial heuristic rules in more promising search directions.

Individual Co-Evolution Reflection: In this step, two randomly selected heuristic strategies are compared, and their performance is evaluated based on test cases. The results are then sent to the reflector LLM, which analyzes the differences and generates suggestions for improvement. SeEvo employs a "language gradient" feedback mechanism to guide the LLM in producing more effective code, with the reward signal being binary (better or worse performance).

Individual Self-Evolution Reflection: For each individual, SeEvo evaluates its performance before and after co-evolution reflection and presents the results to the reflector LLM. The LLM reflects on the changes and offers suggestions for improvement. If performance worsens or remains stagnant, the LLM generates reverse prompts to avoid repeating the same issues. If performance improves, the LLM highlights successful elements and provides enhanced prompts for further optimization.

Collective Evolution Reflection: As SeEvo accumulates experience through multiple iterations, the reflector LLM synthesizes insights from individual self-evolution reflections and individual co-evolution reflections. The goal is to generate prompts that guide the further evolution of heuristic rules. The collective evolution reflection might initially be empty or pre-filled with predefined prompts, but it becomes richer as more iterations are completed.

Crossover: During the crossover step, the LLM generates a new set of heuristic strategies by combining task specifications, performance data of parent strategies, reflections, and detailed generation instructions. Parent heuristic strategies are selected from the population based on their relative performance on test cases, and the LLM integrates insights from mutual reflections to guide the crossover process.

Mutation: The SeEvo method uses an elite mutation strategy, where the LLM generates new heuristic variants by focusing on the best-performing individuals. The mutation prompts include task specifications, elite heuristics, performance reflections, and instructions for generating new solutions.

Individual Evaluation: At both the crossover and muta-

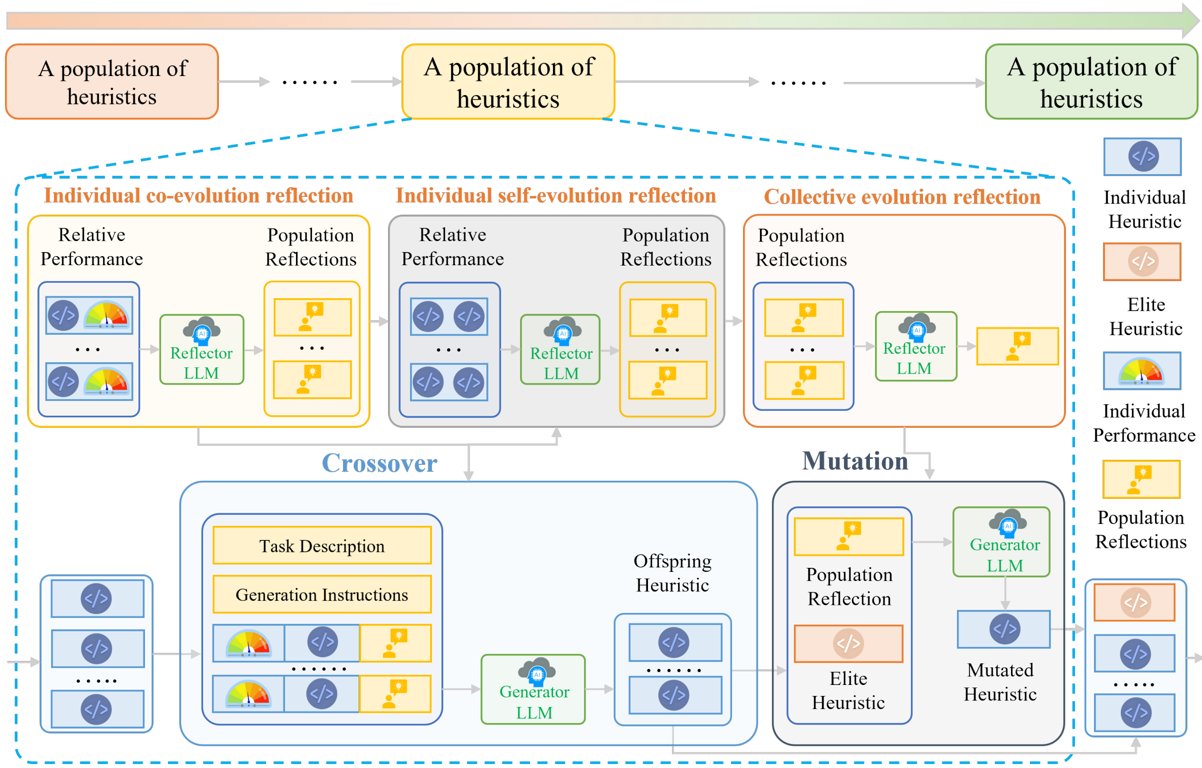

Fig. 2. The population self-evolution method.

tion stages, the effectiveness of each heuristic is rigorously evaluated against the scheduling task. The LLM compares the heuristics against a dataset of test cases, ensuring only the best-performing strategies are preserved. Evaluation at multiple stages helps ensure that each iteration of the evolutionary process refines the population toward better solutions.

In summary, the SeEvo method offers a novel, language-heuristic-based evolutionary approach to solving dynamic scheduling problems, with continuous feedback loops guiding the generation and refinement of heuristic solutions. By incorporating multiple reflection phases, crossover, and mutation, SeEvo ensures an evolving population of heuristics that progressively improves job selection efficiency and performance.

V. Experimental Evaluation

A. Experimental Setup

To validate the effectiveness of the proposed SeEvo method, experiments are conducted under both static and dynamic conditions. For the static experiments, public benchmark datasets from Taillard (TA) and Demirkol (DMU) are used. For the dynamic experiments, various randomly generated DJSSP environments with randomly arriving job orders are simulated. Two APIs, gpt-3.5-turbo-0125 and GLM-3-Turbo, are employed to run the LLM models, considering the high computational cost of gpt-4.0 and previous research findings indicating limited performance improvement with gpt-4.0 [20]. Comparisons are made between SeEvo, GEP [48], multi-tree genetic programming (MTGP) [49], more than ten common HDRs, and three end-to-end DRL methods [17], [18], [50]. Additionally, an ablation study is performed to further validate the robustness of the proposed method.

TABLE I
Parameters of SeEvo

Parameter	Value
LLM (generator and reflector)	gpt-3.5 and GLM-3
LLM temperature (generator and reflector)	1
Population size	20
Maximum number of evaluations	20
Mutation rate	0.5

We use Table I parameters for all SeEvo runs. The SeEvo method is configured with a crossover probability of 1, a mutation probability of 0.5, a population size of 20, and a maximum of 20 generations for training under both static and dynamic conditions. For training, 20 randomly generated cases are used, with one training case replaced after each iteration. The same crossover and mutation probabilities are maintained for testing, but the number of generations is reduced to one for test cases.

In contrast to DRL approaches, the MTGP and GEP methods are more tailored to perform well on specific cases and rely more heavily on training data [13]. However, using test cases directly as training data would create an unfair comparison with other methods. To address this, following the approach of [13], 80 training cases are introduced for the self-generated algorithm, with each round consisting of 20 generations. After each round, the training cases are replaced. The population size for GP and GEP is set to 20, matching that of SeEvo, and four randomly selected cases are used per round, with

TABLE II
EXPERIMENTAL RESULTS ON DMU BENCHMARK, WHERE THE "UB" COLUMN INDICATES THE BEST-KNOWN SOLUTION

Cases	Size	Random	LPT	SPT	STPT	MPSR	DRL-Liu	GP	GEP	SeEvo(GLM3)	SeEvo(GPT3.5)	UB
DMU03	20×15	3827	4592	3630	4232	3435	3303	3540	3651	3462	**3238**	2731
DMU04	20×15	3889	4047	3541	4642	3355	3321	3406	3499	3235	**3212**	2669
DMU08	20×20	4228	4551	4714	4459	3999	4098	3802	4023	**3728**	**3728**	3188
DMU09	20×20	4094	4511	4283	4690	3869	**3753**	4196	4136	3857	3828	3092
DMU13	30×15	5451	5580	4813	5207	4759	4708	4765	4812	**4658**	4709	3681
DMU14	30×15	5306	5591	4583	4811	4238	4124	4289	4213	**3980**	**3980**	3394
DMU18	30×20	5326	5810	6231	5480	5003	4800	**4696**	4917	4724	4724	3844
DMU19	30×20	5174	5787	5126	5203	4930	4837	**4666**	5245	4715	4816	3768
DMU23	40×15	5948	7045	6250	6521	5383	5240	5391	5595	**5151**	5258	4668
DMU24	40×15	6078	6484	5503	6595	5358	5319	5560	5458	**5226**	5316	4648
DMU28	40×20	6737	7322	6558	7697	5927	5948	6017	6142	**5838**	5944	4692
DMU29	40×20	6602	7386	6565	7690	6107	**5824**	6236	6224	5941	5825	4691
DMU33	50×15	6890	8779	7361	7631	6282	6458	6109	6081	**6029**	**6029**	5728
DMU34	50×15	7523	7991	7026	7740	6359	6284	6327	6279	6148	**6146**	5385
DMU38	50×20	7685	9051	7954	8555	7604	7275	7267	7501	**7168**	7170	5713
DMU39	50×20	8097	8514	7592	8908	6953	6776	6941	7124	6693	**6590**	5747
Mean		5803.44	6440.06	5733.13	6253.81	5222.56	5129	5200.50	5306.25	5034.56	**5032.06**	4227.44

parameters aligned with the existing literature [48], [49]. All methods are implemented in Python and executed on a server with an Intel(R) Xeon(R) W-3365 CPU @ 2.70GHz running Ubuntu 20.04.

B. Generalization Performance on Public Benchmark Datasets

In this section, the generalization performance of SeEvo is first evaluated using randomly generated training cases, ranging in size from 20 to 100 jobs and 10 to 20 machines, with processing times for each job randomly selected between 50 and 100 units. For the test sets, 32 benchmark cases from the TA and DMU datasets are selected, covering 8 different problem sizes. The TA test set sizes range from 15×15 to 100×20, while the DMU test set sizes range from 20×15 to 50×20.

Performance on the DMU Benchmark: This section compares five well-known HDRs, including Random selection, longest processing time (LPT), shortest processing time (SPT), shortest total processing time (STPT), and most process sequence remaining (MPSR), along with MTGP, GEP, DRL [18], and SeEvo methods. The results, shown in Table II, use data for the heuristic scheduling rules and DRL-Liu sourced from the literature [18]. The proposed SeEvo method outperforms the other approaches in 12 of 16 test cases. In the remaining 4 cases, SeEvo ranks second, slightly behind the best-performing method. These results demonstrate that SeEvo can effectively schedule and generalize across JSSP cases of different sizes, showcasing strong generalization capabilities. Additionally, GLM-3 and GPT-3.5 each show strengths and weaknesses across different cases, with minimal overall performance differences between the two models.

Performance on the TA Benchmark: In addition to the DMU benchmark experiments, the performance of SeEvo is also evaluated on the TA benchmark datasets. Comparisons include three recent DRL-based approaches (DRL-Chen [17], DRL-Zhang [50], DRL-Liu[18]) and three heuristic dispatching rules: longest processing time for subsequent operations (LSO), shortest processing time$\times$minimum total working time (SPT$\times$TWK), and shortest processing time/minimum total working time remaining (SPT/TWKR). MTGP and GEP are also included for comparison. The results, presented in Table III, source heuristic scheduling rule data from [17] and DRL data from [18]. As shown in Table III, SeEvo outperforms other approaches in 14 of the 16 test cases, ranking second in the remaining 2 cases. These results emphasize the superior optimization capabilities of SeEvo across a variety of scheduling scenarios.

Notably, across both DMU and TA benchmark datasets, SeEvo demonstrates consistent and superior performance in terms of optimization results, indicating that the method is

TABLE III
EXPERIMENTAL RESULTS ON TA BENCHMARK, WHERE THE "UB" COLUMN INDICATES THE BEST-KNOWN SOLUTION

Cases	Size	LSO	SPT/TWKR	DRL-Chen	DRL-Zhang	DRL-Liu	GP	GEP	SeEvo(GLM3)	SeEvo(GPT3.5)	UB
TA01	15×15	1957	1664	1711	1433	1492	1547	1547	**1427**	**1427**	1231
TA02	15×15	1759	1538	1639	1544	**1425**	1565	1486	1465	1437	1244
TA11	20×15	2216	1886	1833	1794	1752	1749	1819	**1656**	1692	1357
TA12	20×15	2187	1969	1765	1805	1692	1789	1732	1637	**1616**	1367
TA21	20×20	2647	2206	2145	2252	2097	2090	2089	**1977**	2007	1642
TA22	20×20	2522	2111	2015	2102	1924	2059	1981	**1915**	1973	1600
TA31	30×15	2478	2435	2382	2565	2277	2307	2279	2192	**2184**	1764
TA32	30×15	2634	2512	2458	2388	**2203**	2400	2396	2279	2274	1784
TA41	30×20	2873	2898	2541	2667	2698	2851	2729	2543	**2496**	2005
TA42	30×20	3096	2813	2762	2664	2623	2603	2613	**2363**	2454	1937
TA51	50×15	3844	3768	3762	3599	3608	3603	3668	**3364**	3412	2760
TA52	50×15	3715	3588	3511	3341	3524	3346	3324	3286	**3245**	2756
TA61	50×20	4188	3752	3633	3654	3548	3685	3642	**3529**	3537	2868
TA62	50×20	4217	3925	3712	3722	3557	3636	3723	**3446**	3474	2869
TA71	100×20	6754	6705	6321	6452	6289	6305	6278	**6071**	6099	5464
TA72	100×20	6674	6351	6232	5695	6002	5776	5625	5604	**5575**	5181
Mean		3360.06	3132.56	3026.38	2979.81	2919.44	2956.94	2933.19	**2797.13**	2806.38	2364.31

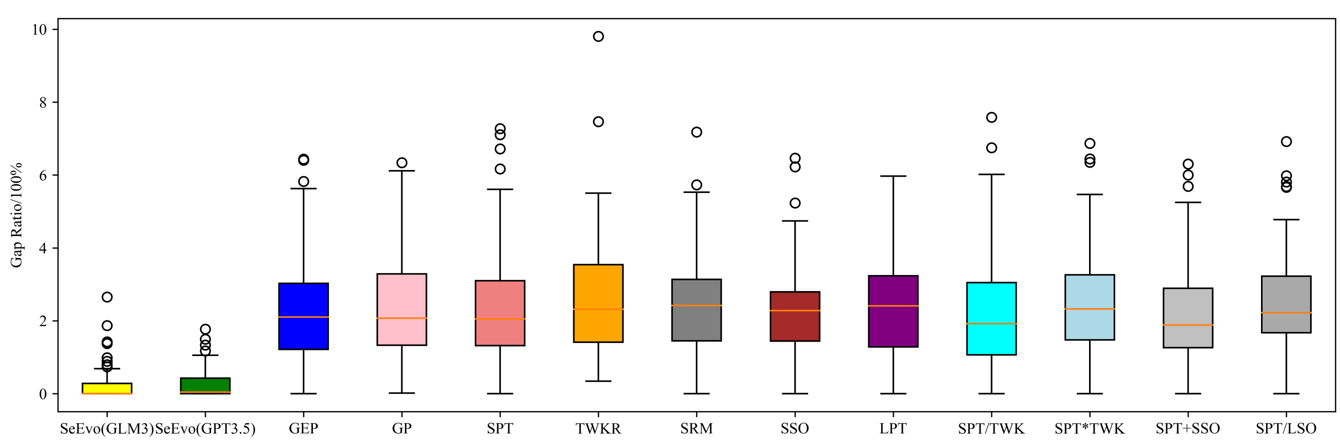

Fig. 3. Gap Ratio of Different Scheduling Methods in Dynamic Cases.

capable of generating high-quality scheduling decisions for various JSSPs. This is true even when problem sizes and processing times differ significantly from the training cases, underscoring the strong generalization ability of SeEvo.

Additionally, the results further validate the LLM's ability to understand and generate high-quality heuristic rules. The strong performance of the SeEvo method is attributed to the effective prompt engineering and domain knowledge gained from 20 training sessions, allowing the model to generate high-quality scheduling decisions efficiently. For instance, on a test set with 50 jobs and 15 machines, GPT-3.5 produces a high-quality solution in just 19 seconds, while GLM-3 requires only 29 seconds to generate a similarly high-quality result.

C. Performance Evaluation in Dynamic Cases with Randomly Arriving Orders

In this section, the performance of the SeEvo method is evaluated in a dynamic environment characterized by the uncertainty of randomly arriving job orders. To simulate this, 20 training cases are generated, with order sizes in each batch ranging between 20 and 50 jobs across 2 to 3 batches. The arrival times for each batch follow a uniform distribution within the intervals [1, 500] and [501, 1000]. The test environment is configured with 10 machines, and the job processing times are random integers between 50 and 100 units. For the test set, 100 cases are generated using the same method as the training set. The results are displayed in Fig.3, which compares SeEvo with 9 common HDRs, including SPT, TWKR, shortest remaining machining time (SRM), shortest subsequent operation (SSO), LPT, LPT/TWK, SPT×TWK, SPT+SSO, and SPT/LSO, as well as MTGP and GEP. Fig.3 illustrates the gap ratio between each method and the optimal result across 13 methods for each case.

As shown in Fig.3, SeEvo consistently demonstrates clear advantages under dynamic conditions, with significant performance improvements in most cases. The proposed method exhibits minimal relative gaps, with the maximum gap ratio not exceeding 30% and, in most cases, staying below 10%. These results indicate that the SeEvo method is effective in achieving promising scheduling outcomes across various DJSSP cases, even in dynamic environments. Furthermore, SeEvo has proven highly capable of handling unseen dynamic environments and uncertainties, which closely resemble real-world production shop conditions.

We attribute the promising performance of the proposed method to several factors. First, the language-heuristic framework is designed to integrate rich domain knowledge, enabling the generation of highly generalized HDRs and prompts. These HDRs are derived from training sets of 20 randomly sized order batches, enhancing the model's ability to generalize across new scenarios. As a result, the method can generate effective decisions with just a single iteration. Second, the SeEvo method enhances exploration and exploitation capabilities through individual co-evolution reflection, individual self-evolution reflection, and collective evolution reflection, ensuring the high quality of the generated HDRs.

D. Ablation Study

The core innovation of the proposed SeEvo method is the introduction of individual self-evolution. To assess the impact of this feature, an ablation study is conducted by removing the individual self-evolution process, with the modified method referred to as ReEvo. The exploration and exploitation capabilities of both SeEvo and ReEvo are evaluated by iterating 50 times on static training sets, followed by testing the generalization performance of the methods.

First, training is performed on the datasets from Table II and Table III, with the convergence curves of these iterations displayed in Fig.4. The x-axis represents the number of validation iterations, and the y-axis shows the makespan (the average makespan for each dataset). As shown in Fig.5, ReEvo, which lacks individual self-evolution, significantly underperforms compared to SeEvo, even exhibiting worse performance than GP on the DMU dataset. This highlights that incorporating individual self-evolution enhances the method's exploration capabilities.

Additionally, the generalization performance of ReEvo on the TA test set is examined, with two different problem sizes: 30 jobs with 20 machines and 50 jobs with 20 machines. Each problem size includes 10 cases, and the optimization goal is to minimize the makespan. The results, shown in Table IV, indicate that SeEvo (GLM-3) significantly outperforms both ReEvo and two end-to-end DRL methods [18]. This confirms that individual self-evolution is crucial in enhancing the method's ability to explore and exploit solution spaces.

Overall, SeEvo (GLM-3) achieves better average results than gpt-3.5, primarily due to the clearer heuristic strategies generated by GLM-3-Turbo. Fig.5 illustrates the HDRs generated by the two API models, with the lighter top half representing HDRs produced by GLM-3, and the darker section representing those generated by gpt-3.5.

VI. Discussion

To address the challenges of poor generalization and reliance on random search in automatic algorithm design for the DJSSP, this paper proposes a dynamic evolutionary framework leveraging LLMs. Specifically, the innovative SeEvo method is introduced and thoroughly validated through both static and dynamic case studies. The experimental results demonstrate that the proposed SeEvo method significantly outperforms commonly used HDRs, MTGP, GEP, and end-to-end DRL methods in terms of static generalization performance. Additionally, in dynamic experiments, SeEvo exhibits substantial advantages in most cases. These findings confirm the capability of LLMs to generate highly targeted and adaptable HDRs, benefiting from their robust language understanding, generation capabilities, and extensive domain knowledge acquired through training on diverse datasets.

Despite these promising results, this study has certain limitations. The LLMs are used via scheduling APIs for training and validation without direct fine-tuning for specific DJSSP knowledge. Moreover, only a single HDR is employed in the dynamic cases, which limits the method's performance, preventing it from achieving the absolute superiority observed

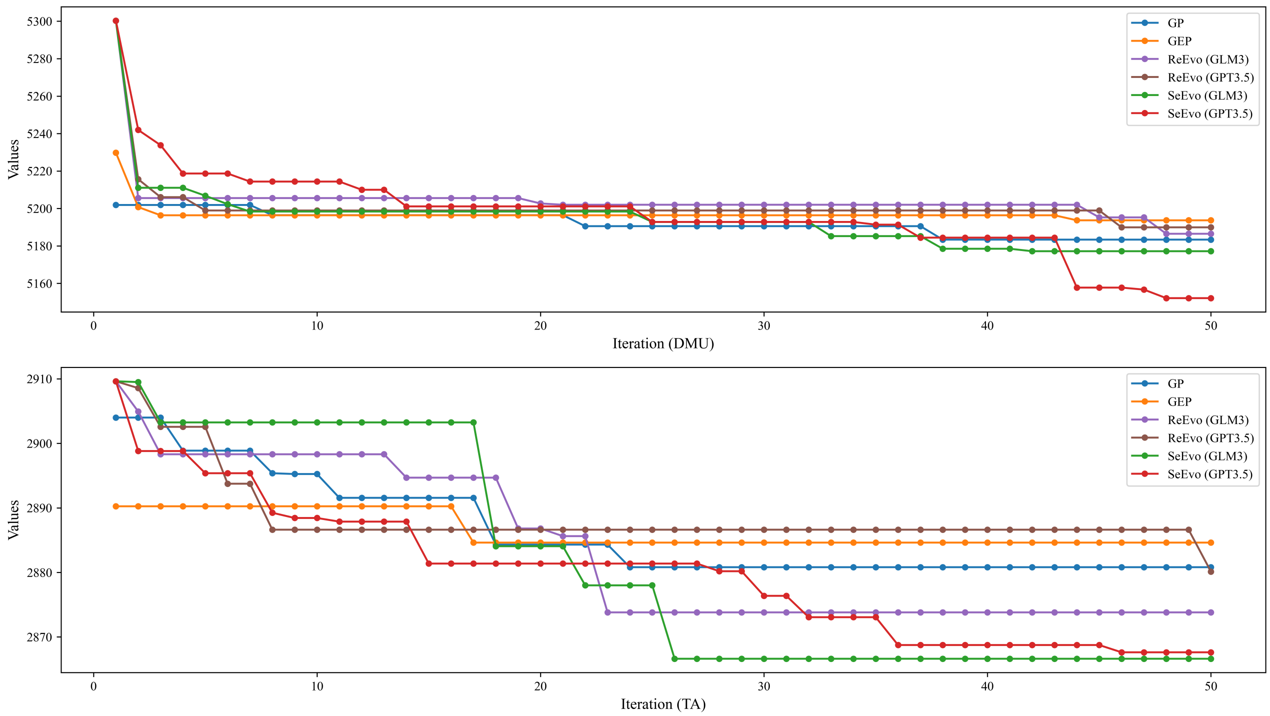

Fig. 4. Convergence Curves of Different Methods on Two Training Case Groups.

TABLE IV
ABLATION STUDY RESULTS

Instance	DRL-GAT	DRL-GCN	SeEvo(GLM3)	SeEvo(GPT3.5)	ReEvo(GLM3)	ReEvo(GPT3.5)
30x20	2628	2740	**2445**	2461	2452	2456
50x20	3547	3703	**3352**	3367	3372	3387

```
def get_combined_expression_v2(pt: np.ndarray, wkr: np.ndarray, rm: np.ndarray, so:
np.ndarray, twk: np.ndarray) -> np.ndarray:
    # Calculate the normalized processing time difference
    pt_diff_norm = (pt - wkr) / twk

    # Calculate the product
    rm_so_product = rm * so

    # Create an array for the combined expression
    combined_expression_data = np.empty_like(pt)

    # Adjust the weights dynamically based on the current instance
    # Prioritize reducing the makespan by giving more weight to it
    makespan_weight = 1.5

    # Simplify the expression by focusing on the most impactful variables
    combined_expression_data = pt_diff_norm - rm_so_product

    # Apply a dynamic weight to emphasize reducing the makespan
    combined_expression_data *= (1 + makespan_weight * pt_diff_norm)

    return combined_expression_data
```

```
import numpy as np
def get_combined_expression_v2(pt: np.ndarray, wkr: np.ndarray, rm: np.ndarray, so:
np.ndarray, twk: np.ndarray) -> np.ndarray:
    combined_expression_data = np.sin(((pt) + np.sin(rm)))
    return combined_expression_data
```

Fig. 5. HDRs generated by Two API Models on TA70 Case.

in the static cases. Future research will focus on two key areas: (1) improving the generalization performance in static cases by integrating vector databases and employing LLM fine-tuning techniques to enhance the model's scheduling knowledge, and (2) enhancing adaptability to dynamic environments by utilizing a combination of multiple HDRs to further optimize performance.

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